
UNIT CONVERSIONS

❑ ***THE BASIC FACTS OF LENGTH IN THE U.S. SYSTEM:
INCHES (IN), FEET (FT), YARDS (YD), AND MILES (MI)***

12 in = 1 ft
3 ft = 1 yd
5,280 ft = 1 mi

Note that the smallest of the four units is inch, while the largest is mile. The word **inch** comes from the Latin word *unicia*, which means “twelfth part.”

This system, along with units like ounces, tons, quarts, and gallons is sometimes called the U.S. system, since we’re the only industrialized country in the world that still uses it.

❑ ***FOUR BASIC FACTS FROM ARITHMETIC***

#1: Any quantity divided by 1 is itself. For example,

$$\frac{89}{1} = 89 \qquad \frac{x^2}{1} = x^2 \qquad \frac{8\text{yd}}{1} = 8\text{yd}$$

#2: Any quantity (except 0) divided by itself is 1. For instance,

$$\frac{23}{23} = 1 \qquad \frac{\pi}{\pi} = 1 \qquad \frac{12\text{ inches}}{1\text{ foot}} = 1$$

#3: Any quantity multiplied by 1 is itself.

$$\frac{23}{30} \times 1 = \frac{23}{30}$$

$$N \cdot 1 = N$$

$$15 \times \underbrace{\frac{3 \text{ ft}}{1 \text{ yd}}}_{= 1} = 15$$

#4: We recall the procedure of “cross-canceling” when we multiply fractions:

$$\frac{8}{7} \times \frac{3}{8} = \frac{\cancel{8}}{7} \times \frac{3}{\cancel{8}} = \frac{3}{7}$$

This cross-canceling even works for units of measurement:

$$\frac{7 \text{ ft}}{1} \times \frac{12 \text{ in}}{1 \text{ ft}} = \frac{7 \cancel{\text{ft}}}{1} \times \frac{12 \text{ in}}{1 \cancel{\text{ft}}} = \frac{84 \text{ in}}{1} = 84 \text{ in}$$

This example is a preview of the task coming up, and shows you how to convert 7 ft into an equivalent number of inches.

Terminology: We call fractions like $\frac{12 \text{ inches}}{1 \text{ foot}}$ and $\frac{3 \text{ ft}}{1 \text{ yd}}$ **conversion factors**; the top and the bottom of the fraction are equal to each other, and therefore each fraction has the value 1. Can you explain why $\frac{1 \text{ mi}}{5,280 \text{ ft}}$ is a conversion factor? Notice that by flipping over each of these three conversion factors, we get three more conversion factors, for a total of six. It’s these six conversion factors that are the basis of our method for converting between units of length in the U.S. system:

$$\frac{12 \text{ inches}}{1 \text{ foot}} \quad \frac{3 \text{ ft}}{1 \text{ yd}} \quad \frac{1 \text{ mi}}{5,280 \text{ ft}} \quad \frac{1 \text{ foot}}{12 \text{ inches}} \quad \frac{1 \text{ yd}}{3 \text{ ft}} \quad \frac{5,280 \text{ ft}}{1 \text{ mi}}$$

Each of these fractions is a *conversion factor*; the numerator and denominator are equal, and thus each fraction is equal to 1.

❑ **CONVERSION EXAMPLES**

There are many ways to convert from one unit of measurement to another. The method we'll use here is a little tricky at first, but it works beautifully in chemistry and other courses. In fact, the less you understand the units of measurement, the more valuable this method becomes.

EXAMPLE 1: Convert 30 inches to feet.

Solution:

Step 1: Put the given problem over 1: $\frac{30 \text{ in}}{1}$

Step 2: To get rid of the inch unit, multiply by the conversion factor that will cancel that unit, $\frac{1 \text{ ft}}{12 \text{ in}}$:

$$\frac{30 \text{ in}}{1} \times \frac{1 \text{ ft}}{12 \text{ in}}$$

Step 3: Now we cross-cancel the inches, do the arithmetic, leaving just the unit feet, our goal:

$$\frac{30 \cancel{\text{ in}}}{1} \times \frac{1 \text{ ft}}{12 \cancel{\text{ in}}} = \frac{30 \text{ ft}}{12} = \mathbf{2.5 \text{ ft}}$$

EXAMPLE 2: Convert 3.2 miles to yards.

Solution: We have no basic fact that directly connects miles and yards, but we can use feet as an intermediary, or in-between, step.

Step 1: Put the given problem over 1:

$$\frac{3.2 \text{ mi}}{1}$$

Step 2: To rid ourselves of miles, multiply by the conversion factor $\frac{5,280 \text{ ft}}{1 \text{ mi}}$:

$$\frac{3.2 \text{ mi}}{1} \times \frac{5,280 \text{ ft}}{1 \text{ mi}}$$

Step 3: Though the miles will cross-cancel, at this point we'd be at feet, not at our goal of yards. So, you know what? We tack on the additional conversion factor $\frac{1 \text{ yd}}{3 \text{ ft}}$; this will eliminate the feet and leave us with yards:

$$\frac{3.2 \text{ mi}}{1} \times \frac{5,280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ yd}}{3 \text{ ft}}$$

Step 4: Now we cancel the miles, cancel the feet, do the arithmetic, and we're left with nothing but yards:

$$\frac{3.2 \cancel{\text{mi}}}{1} \times \frac{5,280 \cancel{\text{ft}}}{1 \cancel{\text{mi}}} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} = \frac{16,896 \text{ yd}}{3} = \mathbf{5,632 \text{ yd}}$$

Homework

Using a calculator if you like, convert and round to the nearest hundredth:

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|---------------------|---------------------|-----------------|
| 1. 5 mi to yd | 2. 4,224 ft to mi | 3. 880 yd to mi |
| 4. 90 in to yd | 5. 1.7 yd to in | 6. 2.4 mi to ft |
| 7. 30 ft to yd | 8. 2.7 ft to in | 9. 30 in to ft |
| 10. 1 mi to in | 11. 12,672 in to mi | 12. 11 in to ft |
| 13. 15,840 ft to mi | 14. 13 mi to yd | 15. 44 ft to yd |
| 16. 66 in to yd | 17. 1.5 yd to in | 18. 2 mi to in |

- | | | |
|------------------|------------------|---------------------|
| 19. 0.9 ft to in | 20. 0.7 mi to ft | 21. 0.5 yd to ft |
| 22. 33 in to yd | 23. 1.7 yd to ft | 24. 1.5 mi to in |
| 25. 14 in to ft | 26. 7 ft to yd | 27. 32 mi to yd |
| 28. 1.7 mi to ft | 29. 21 ft to yd | 30. 16,352 yd to mi |
| 31. 13 in to ft | 32. 1.9 yd to ft | 33. 69,696 in to mi |

❑ A GIANT LENGTH UNIT

An **astronomical unit** (AU) is the average distance from the Earth to the Sun, about 92,956,000 miles. This unit of length helps simplify calculations. The distance from the Sun to Pluto, for example, is 39.53 AU, which is a much easier number to work with compared to 3,674,590,210 miles. A **light-year** is even a longer distance — a much longer distance. Look it up.



❑ ANOTHER INSTANCE OF CROSS-CANCELING UNITS

You may have learned that multiplying the average rate (speed) that an object travels by the amount of time it travels gives us the distance it travels:

$$rt = d$$

For instance, at a speed of 32 mph and a time of 5 hr, the distance is

$$rt = (32 \text{ mph})(5 \text{ hr}) = 160 \text{ mi}$$

But instead of writing “miles per hour” as mph, we can write it more mathematically as $\frac{\text{mi}}{\text{hr}}$. Now the same problem can be written like this:

$$rt = 32 \frac{\text{mi}}{\text{hr}} \times 5 \text{ hr} = 32 \frac{\text{mi}}{\text{hr}} \times 5 \frac{\text{hr}}{1} = 32 \frac{\text{mi}}{\cancel{\text{hr}}} \times 5 \frac{\cancel{\text{hr}}}{1} = 160 \text{ mi}$$

We get the final unit in miles, just as distance (in this problem) should be, by cross-canceling the unit “hr”.

❑ ***THE BASIC FACTS OF LIQUID VOLUME IN THE U.S. SYSTEM:
FLUID OUNCES (FL OZ) CUPS (C), CINTS (PT), QUARTS
(Q), AND GALLONS GAL)***

$8 \text{ fl oz} = 1 \text{ c}$
$2 \text{ c} = 1 \text{ pt}$
$2 \text{ pt} = 1 \text{ qt}$
$4 \text{ qt} = 1 \text{ gal}$

The word **gallon** comes from the Latin word *galleta*, meaning “bucket.” And the term **cup** comes from the Latin word *cupa*, meaning “tub.”



Here are some conversion factors that we will need in the homework:

$$\frac{1 \text{ gal}}{4 \text{ qt}}$$

$$\frac{1 \text{ qt}}{2 \text{ pt}}$$

$$\frac{2 \text{ c}}{1 \text{ pt}}$$

$$\frac{8 \text{ fl oz}}{1 \text{ c}}$$

Remember that each fraction is a conversion factor because the numerator and the denominator are equal to each other (for example, $1 \text{ gal} = 4 \text{ qt}$). This, of course, means that each conversion factor is ultimately equal to 1.

To help you understand that weight and volume are completely different concepts, consider



that a volleyball and a bowling ball have about the same volume (they’re about the same size), but they certainly do not have the same weight.



Conversion Examples:**EXAMPLE 1: Convert 100 cups to quarts.**

Solution: Cups are smaller than quarts, so we expect that 100 cups will produce a smaller number of quarts. Using the appropriate conversion factors, we proceed as follows:

$$\frac{100 \cancel{c}}{1} \times \frac{1 \cancel{pt}}{2 \cancel{c}} \times \frac{1 qt}{2 \cancel{pt}} = \frac{100}{4} qt = \mathbf{25 qt}$$

EXAMPLE 2: Convert 3.4 gallons to fluid ounces.

Solution: Gallons are big compared to fluid ounces, so we expect that 3.4 gallons will convert to a whole lot of fluid ounces.

$$\frac{3.4 \cancel{gal}}{1} \times \frac{4 qt}{1 \cancel{gal}} \times \frac{2 \cancel{pt}}{1 qt} \times \frac{2 \cancel{c}}{1 \cancel{pt}} \times \frac{8 fl oz}{1 \cancel{c}} = \mathbf{435.2 fl oz}$$

Homework

Convert and round to the nearest hundredth, using a calculator if you like:

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|---------------------|----------------------|---------------------|
| 34. 7.1 qt to pt | 35. 8.8 pt to c | 36. 5.5 gal to c |
| 37. 6.7 gal to qt | 38. 24 c to gal | 39. 68 fl oz to pt |
| 40. 33 c to fl oz | 41. 2.7 qt to c | 42. 0.9 qt to fl oz |
| 43. 6.6 gal to pt | 44. 0.7 gal to fl oz | 45. 71 pt to qt |
| 46. 1.2 pt to fl oz | 47. 4.9 c to pt | 48. 5.2 pt to c |
| 49. 34 fl oz to c | 50. 407 pt to qt | 51. 3.4 qt to gal |
| 52. 55 c to fl oz | 53. 0.6 gal to fl oz | 54. 60 fl oz to pt |

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|---------------------|-------------------|------------------|
| 55. 8.8 gal to qt | 56. 2.8 c to pt | 57. 7.5 qt to c |
| 58. 0.9 qt to fl oz | 59. 484 pt to gal | 60. 9.5 gal to c |
| 61. 1.9 pt to fl oz | 62. 8.8 gal to pt | 63. 3.7 qt to pt |
| 64. 2.5 gal to pt | 65. 117 pt to qt | 66. 2.3 c to pt |
| 67. 4.1 gal to c | 68. 4.5 qt to gal | 69. 2.8 c to qt |

Solutions

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|---------------|---------------|----------------|
| 1. 8,800 yd | 2. 0.8 mi | 3. 0.5 mi |
| 4. 2.5 yd | 5. 61.2 in | 6. 12,672 ft |
| 7. 10 yd | 8. 32.4 in | 9. 2.5 ft |
| 10. 63,360 in | 11. 0.2 mi | 12. 0.92 ft |
| 13. 3 mi | 14. 22,880 yd | 15. 14.67 yd |
| 16. 1.83 yd | 17. 54 in | 18. 126,720 in |
| 19. 10.8 in | 20. 3,696 ft | 21. 1.5 ft |
| 22. 0.92 yd | 23. 5.1 ft | 24. 95,040 in |
| 25. 1.17 ft | 26. 2.33 yd | 27. 56,320 yd |
| 28. 8,976 ft | 29. 7 yd | 30. 9.29 mi |
| 31. 1.08 ft | 32. 5.7 ft | 33. 1.1 mi |
| 34. 14.2 pt | 35. 17.6 c | 36. 88 c |
| 37. 26.8 qt | 38. 1.5 gal | 39. 4.25 pt |
| 40. 264 fl oz | 41. 10.8 c | 42. 28.8 fl oz |

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|-----|------------|-----|------------|-----|----------|
| 43. | 52.8 pt | 44. | 89.6 fl oz | 45. | 35.5 qt |
| 46. | 19.2 fl oz | 47. | 2.45 pt | 48. | 10.4 c |
| 49. | 4.25 c | 50. | 203.5 qt | 51. | 0.85 gal |
| 52. | 440 fl oz | 53. | 76.8 fl oz | 54. | 3.75 pt |
| 55. | 35.2 qt | 56. | 1.4 pt | 57. | 30 c |
| 58. | 28.8 fl oz | 59. | 60.5 gal | 60. | 152 c |
| 61. | 30.4 fl oz | 62. | 70.4 pt | 63. | 7.4 pt |
| 64. | 20 pt | 65. | 58.5 qt | 66. | 1.15 pt |
| 67. | 65.6 c | 68. | 1.125 gal | 69. | 0.7 qt |

“The highest activity a human being can attain is learning for understanding, because to understand is to be free.”

– Baruch Spinoza